# Kinematics

Kinematics is the study of motion. Generally, this involves describing the position, velocity, and acceleration of an object.

### **Reference frame**

In order to describe movement, we need to set a reference frame. This reference frame allows us to use a set of coordinates to refer to the position of an object at any point in time. For example, we could set a reference frame around a map of the United States. By doing this, we could then identify the location of any city by a set of (x, y) coordinates.



If we set our camera up correctly, we can set a reference frame to analyze the kinematics of movements such as a 100 m run.



In these examples, we are setting a 'global' reference frame – one that does not move. In the 100 m run example, we could set the origin of the reference frame at the start line (0 m mark). Then, for example, we could identify the time that the runner crosses the 10 m, 20 m, etc. mark.

We can also set the origin of the x,y plot in a convenient location and direction. In the picture below, the origin is set at the bottom right with positive direction of movement headed in the left direction.



The critical aspect of setting a reference x frame is defining the origin and the direction of each axis.

#### Motion

Motion is defined as an object's (or person's) change in space over some time period. There are three types of motion:

- 1. Linear
- 2. Angular
- 3. General

Linear motion is defined as when all points of an object travel the same distance. Angular motion is defined as when all points of an object travel the same angular distance. General motion is defined as motion that consists of both linear and angular motion. Distance is the measure of the total change in position of an object or point on an object.

Linear Motion

There are two types of linear motion:

- 1. Rectilinear: motion in a straight path.
- 2. Curvilinear: motion about a curved path.

**Rectilinear Motion:** 

Curvilinear Motion:

Note that in both types of linear motion, the segment maintains the same orientation in space and all points of the object travel the same distance.

#### Angular Motion

Note that the segment rotates about some axis and all points travel the same angular distance.

## Linear Kinematics

Position

Position is a parameter used to describe an object's change in space. Two parameters can be used to describe position:

- 1. Distance: The total change in position.
- 2. Displacement: The straight line distance between start and finish positions  $\Delta s=s_{f}-s_{i}$  units = any units of length (e.g., meters, feet, yards, miles, etc.).

Distance is referred to as a 'scalar' while Displacement is referred to as a 'vector.'

Scalar: A quantity that is fully described by magnitude only. Vector: A quantity that is described by magnitude and direction.

Consider that when we discuss vector quantities, the math of functions such as addition, subtraction, multiplication, division and other math functions involves some special math rules. These rules will be discussed when necessary. At this time, you should be aware that there are some special rules to follow when we are dealing with vector quantities.

As vectors are presented and discussed, we will typically describe motion in one direction only (e.g., horizontal or vertical). When we do that, the vector direction will be noted by whether the parameter is positive or negative whereas the magnitude is the absolute value of the parameter. For example, if displacement is +1 m, the direction of the vector is positive and the magnitude is 1 m. Likewise, if displacement is -1 m, the direction of the vector of the vector is negative and the magnitude is 1 m.

#### Velocity

Velocity is the measure of how fast an object is changing position. Specifically, velocity is calculated as the rate of change of displacement.

 $v = \triangle s / \triangle t$  units = units of displacement over units of time (e.g., m/s, ft/min, etc.)

Speed is also a measure of how fast an object is changing position, but it is calculated as the rate of change of distance.

Velocity is a vector (special math) and speed is a scalar.

When we analyze movement in one direction (e.g., horizontal or vertical), we can tell the direction of movement by knowing if velocity is positive or negative. For example, let's consider running a 100 m run. If velocity was +10 m/s, we know the runner is moving towards the finish line (movement is in the positive direction). If velocity was -10 m/s, that would simply mean the runner is moving away from the start line ... and in the wrong direction! In both those examples, it is important to note that the runner was running at the same rate (i.e., 10 m/s) ... just in different directions.

The direction of velocity tells us the direction of movement.

Acceleration

Acceleration is the measure the rate of change of velocity. That is, acceleration describes how quickly velocity is changing (we will not discuss the rate of change of speed).

 $a = \Delta v / \Delta t$  units = units of velocity over units of time (e.g., m/s/s = m/s<sup>2</sup>)

...and remember, whenever calculating  $\triangle$  (a change), it is always *final* – *initial*.

When describing acceleration, we will use the following statement:

Velocity is (choose one: speeding up, slowing down, constant) while movement is (in the positive or negative direction).

For example, if velocity changed from 2 m/s to 3 m/s, we first should recognize that velocity increased (i.e., 3 m/s is faster than 2 m/s). So, we know that velocity is speeding up. Next, we need to know what direction movement is in. You can easily do that by looking at the direction of velocity. In this example, velocity is positive and therefore we could say "Velocity is speeding up while movement is in the positive direction."

Let's consider another example where velocity changed from -2 m/s to -3 m/s. In this example, the magnitude of the velocity (i.e., how fast movement is occurring) is the absolute value of the velocity vector (i.e., 2 m/s and 3 m/s). So, we know that velocity is speeding up (i.e., going faster). The direction of the movement can be determined by the direction of velocity. In this example, velocity is negative and therefore the direction of movement is negative and we would say: Velocity is speeding up while movement is in the negative direction.

#### **Position – Velocity – Acceleration**

The kinematic parameters, PVA, are related mathematically. Using calculus terms, velocity is the time derivative of position and acceleration is the time derivative of velocity. To help us understand these kinematic parameters, we will spend a lot of time graphing how the position of some point changes over time during some activity.

Consider, for example, the following plot of vertical displacement of the Center of Mass (COM) during a squat movement.



Now, consider the formula for velocity:  $v = \Delta s / \Delta t$ 

You may recognize that if you make  $\Delta t$  very small, v represents the slope between  $t_f$  and  $t_i$  of the position vs. time plot (slope = rise/run =  $\Delta y/\Delta x$ ). Considering this observation, the following plot can be constructed for velocity vs. time:



Now, consider the formula for acceleration:  $a = \Delta v / \Delta t$ 

Thus, acceleration represents the slope of the velocity vs. time plot.

Given this information, can you construct the acceleration vs. time plot given the above velocity vs. time plot?

Counter-movement PVA

Construct the Position vs. Time and Acceleration vs. Time plots for the given Velocity vs. Time plot for the counter-movement jump.



#### **Running PVA**

You should understand that the significance of a negative or positive velocity is understood by inspection of the position vs. time plot. Likewise, the significance of a negative or positive acceleration is understood by inspection of the velocity vs. time plot. The reason for this is that the positive or negative sign indicates the direction of a vector. Remember, we are dealing with the vertical direction only, and you can move upwards or downwards (+ or -) in the vertical direction. To understand what the + or – sign means, knowledge of initial conditions is necessary.

Consider the vertical position vs. time plot for the COM during the stance phase of running (i.e. when the foot is in contact with the ground).

Construct the vertical velocity vs. time plot starting the plot at -1m/s (i.e. initial velocity is -1 m/s. Can you explain why this is?).



#### **Practice PVA**

Since the PVA is a very important and powerful tool to understand movement, you should practice constructing different PVA relationships.

Construct the PVA relationship for:

- 1. The **horizontal** PVA during a swimming start. Start your plots with all initial values equal to zero.
- 2. The **vertical** PVA during landing from a jump. Start your plots with initial velocity set to -2 m/s and initial acceleration equal to  $-9.8 \text{ m/s}^2$

On each of the plots, identify critical discrete events. For example, during the swimming start, identify on your plot where the swimmer looses contact with the starting block. During the landing activity, identify where the person contacts the ground.

Use the PVA tool to **understand** what is going on.

#### **Angular Kinematics**

Angular kinematics is the study of angular motion, which is motion about some axis of rotation. Using the parameters of angular position, angular velocity and angular acceleration completes description of angular motion.

Angular position ( $\theta$  'theta') Angular position can be quantified using either

- 1. Absolute angles
- 2. Relative angles

Absolute angles represent the angle between a segment (e.g. the upper arm) and a fixed segment in space. For example, the horizontal axis (of an x,y plot) never moves – therefore, any angle between it and some segment is 'absolute.'



Relative angles represent the angle between two segments, neither of which is fixed in space. For example, the elbow angle is the angle between the upper arm and forearm. The knee angle is the angle between the thigh and leg.



There are distinct advantages and disadvantages to using each system. We will start our angular kinematics adventure by examining relative angles.

Angular distance: Total change in angular position. Angular displacement:  $\theta = \theta_f - \theta_i$ . Units: Degrees Radians Revolutions

Angular velocity ( $\omega$  'omega')

Rate of change of angular displacement.  $\omega = \Delta \theta / \Delta t$ If units of degrees are used for  $\theta$ , angular velocity is in degrees/s.

Angular acceleration ( $\alpha$  'alpha') Rate of change of angular velocity  $\alpha = \Delta \omega / \Delta t$  units = deg/s<sup>2</sup>

#### **Angular PVA**

The relationship between angular position, velocity and acceleration is identical to that of linear position, velocity and acceleration.

Draw out the angular PVA for the elbow angle during one repetition of a biceps curl exercise. Consider that the exercise starts with the elbow fully extended (180 degrees), flexes to about 30 degrees and then extends again. After constructing a model, enter the Elbow Kinematic data in Appendix D into a spreadsheet, calculate angular velocity and acceleration, and then plot out each parameter.

How does your theoretical model compare with the actual recorded data?

Can you explain why there are differences (if any)?

How would your model change if the concentric portion of the exercise were done faster than the eccentric portion?

Relationship between Angular and Linear motion

Now that you have a good understanding of linear and angular motion, the next step is to understand the relationship between these two types of motion. Consider swinging an object around and around, only to let it go and have it fly through the air as a projectile. You intuitively know that to have the object travel far, you simply need to swing the object around faster (i.e., increase angular velocity). Now, consider that the type of motion the object undergoes when in the air is linear motion (yes, there are exceptions to this statement – but it is OK for now!) – clearly there is a relationship between linear and angular motion. That is, the faster you spin the object, the faster it will go upon release. But, that's only part of the story. Consider that the object was on the end of a rope (or, my kids put tennis balls in socks – same idea). Besides increasing angular velocity, can you think of another way to increase the velocity of the object upon release? Yes – increase the length of the rope for a given angular velocity! So, the relationship between angular motion and linear motion can be summarized by a simple equation:

 $v_T = r\omega$ 

where  $v_T$  is the linear velocity (specifically, tangential velocity)

- r is the radius of rotation
- $\omega$  is the angular velocity

The only 'trick' to this relationship is that the units of angular velocity need to be radians per second.